Web-based Supplementary Materials for "Deductive derivation and Turing-computerization of semiparametric efficient estimation"

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December 9, 2014

Appendix C: Template for proving asymptotic normality

In this section we present a template that may be used to prove asymptotic normality of the estimator described in the previous sections. The template follows standard arguments for the analysis of estimators in semiparametric models, but the assumptions need to be verified for every specific parameter τ and every estimation problem. We start by assuming that, for two distributions G and F, the functional τ admits a representation

$$\tau(G) - \tau(F) = \int \phi\{d', (G-\tau), \tau\} dF(d') + R(F-G), \tag{C.1}$$

where $R(F-G)$ is a remainder term, typically a second order term. Such representation will typically hold for parameters that are pathwise differentiable in the sense of Bickel et al. (1993, page 58). Robins et al. (2009) discuss the relation between this representation and the pathwise derivative of τ.

We denote ϵ_n the value used in computation of the numerical Gateaux derivative. In addition to (C.1), under the assumptions:

(i) $\mathsf{Gateaux}\{\tau, F_w(\widehat{\delta}), d', \epsilon_n\}$ belongs to an F-Donsker class with probability tending to one.

(ii) $\int \left[\mathsf{Gateaux}\{\tau, F_w(\widehat{\delta}), d', \epsilon_n\} - \mathsf{Gateaux}\{\tau, F, d', \epsilon_n\}\right]^2 dF(d') \to 0$ in probability,

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(iii) $R(F - F_w(\widehat{\delta})) = o_P(n^{-1/2})$, and

(iv) $\epsilon_n = o(n^{-1/2})$

results from empirical process theory may be used to establish:

$$\widehat{\tau}^{\text{deductive}} - \tau = \frac{1}{n}\sum_{i=1}^{n} \phi\{D_i, (F - \tau), \tau\} + o_P(n^{-1/2}). \tag{C.2}$$

Assumption (i) is a standard assumption and holds if F_w is a parametric model (van der Vaart, 2000, example 19.7), or if it belongs to a class of smooth functions (van der Vaart, 2000, example 19.9), for example. Assumption (ii) is satisfied if $F_w(\widehat{\delta})$ converges to F, and assumption (iii) states the necessary convergence rate. Assumption (iv) guarantees that the error in the approximation of ϕ by the Gateaux derivative is of order $n^{-1/2}$.

References

Bickel, P. J., Klaassen, C. A., Ritov, Y., and Wellner, J. A. (1993). *Efficient and Adaptive Estimation for Semiparametric Models.* Johns Hopkins University Press, Baltimore.

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