## Note:

- 1. Due 11:59PM, October 3<sup>rd</sup>, 2016.
- 2. Electronic submission to your instructor's email.
- 3. You are VERY MUCH encouraged to form teams to discuss proofs and program algorithms. If so, please acknowledge your teammate(s)' contributions at the beginning of your submitted homework. You must independently write your homework based on your own understanding.
- 4. Choose any programming language you like, R, Python, Matlab, C/C++, Julia, etc.

## Theory

**[10 points]** Exercise 2.5, Koller and Friedman (2009). *Remark*: simple useful fact.

[10 points] Exercise 2.9, Koller and Friedman (2009).

*Remark:* Familiarize yourself with semi-graphoid axioms, especially their intuitions in the setting of probabilistic reasoning. Please refer to Section 1.1.5, Pearl (2009) *Causality*.

[15 points] Exercise 3.13, Koller and Friedman (2009).

*Remark*: Convince yourself that Theorem 3.5 in the same book is true.

[15 points] Exercise 7.7, Koller and Friedman (2009).

*Remark:* In Gaussian setting, it derives a formula for pairwise conditional independence, based on which you can draw an undirected graph. Please give a specific numerical example to illustrate this formula.

[*Extra credit:* **15** points] Prove Theorem (Hammersley-Clifford-Besag; Discrete Version). Suppose that G = (V, E) is an undirected graph, where  $X_i \in V, j = 1, ..., d$ , are random variables, each taking on a finite number of values. If  $P(X_1 = x_1, ..., X_d = x_d)$  is strictly positive for arbitrary realized values  $(x_1, ..., x_d)'$  and P satisfies the local Markov property with respect to G, then it factorizes with respect to G.

Remark: "fundamental theorem of random fields".

## **Examples and Implementations**

[25 points] Question 3, Professor Eric Xing's course, as linked below: http://www.cs.cmu.edu/~epxing/Class/10708/hw/pgm\_2014\_homework1.pdf

[25 points; Choose One: a or b]

## BIOSTAT 830 GRAPHICAL MODELS Problem Set 1 – Representations for Graphical Models

- a) Question 4 in the linked pdf above. Access data at: http://www.cs.cmu.edu/~epxing/Class/10708/hw/hw1\_images.mat
- *b)* Exercise 3.28, Koller and Friedman (2009). Run your algorithm on your favorite DAG to demonstrate its correctness.

[Extra credit: 15 points] Exercise 3.11, Koller and Friedman (2009)